THE IMPACT OF RUSSIA–UKRAINE WAR ON BRENT CRUDE OIL MARKET RISK EVALUATIONS

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1. Introduction

Crude oil, a natural resource extracted from underground reservoirs, is a cornerstone of the global energy system due to its diverse applications. It serves as a primary source of energy worldwide and is refined into products such as gasoline, lubricants, and plastics. Beyond its industrial utility, crude oil holds a critical position in financial markets as one of the most actively traded commodities. Its price is closely monitored by investors, policymakers, and firms, given its sensitivity to global economic and geopolitical conditions.

The volatility of crude oil prices is influenced by multiple factors, including supply–demand imbalances, pandemic-induced disruptions, and geopolitical events. Among these, the Russia–Ukraine war, which began in February 2022, stands out as the most consequential driver of oil price volatility in recent years. Sanctions and trade restrictions on Russian exports—including crude oil—disrupted global supply

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chains, amplifying uncertainty in energy markets. As one of the world's leading oil exporters, Russia's restricted output contributed to unprecedented fluctuations: Brent crude experienced both its highest and lowest five-year returns within weeks of the conflict's onset, with a maximum return of 8.1564 percent on March 17, 2022, and a minimum of −13.3124 percent on March 9, 2022.

Such extreme volatility has posed significant challenges for investors, policymakers, and energy-related industries. This underscores the importance of rigorous statistical analysis to better understand crude oil price dynamics under geopolitical stress. Developing robust forecasting models and conducting comprehensive risk assessments can enhance decision-making and mitigate exposure to uncertainty.

The literature review underpinning this research addresses four main themes. First, it examines two foundational theories in finance—the Random Walk Hypothesis and the Efficient Market Hypothesis. Second, it surveys studies on the impact of war and geopolitical shocks on crude oil markets. Third, it reviews econometric research employing ARMA–GARCH and related models to capture crude oil price behavior. Finally, it considers approaches to market risk assessment in the crude oil sector. Together, these bodies of work provide a conceptual and methodological foundation for the present study.

1.1. Random Walk Hypothesis and the Efficient Market Hypothesis: Maiti (2021) highlights two foundational hypotheses in finance and economics: the Efficient Market Hypothesis (EMH) and the Random Walk Hypothesis (RWH). The EMH argues that security prices fully incorporate all available information; consequently, price changes occur only in response to new information, which is inherently unpredictable and thus follows a random process. Similarly, the RWH maintains that individual price changes are independent and random, making reliable forecasting of security prices impossible. Together, these hypotheses suggest that consistently outperforming the market is only possible by taking on higher levels of risk.

The two concepts are closely related. In an efficient market, prices immediately reflect a broad range of information, including news, financial statements, corporate announcements, economic indicators, industry data, and historical trading activity. Since new information arrives randomly, price movements should conform to a random walk. For studies such as ours, which aim to forecast crude oil prices, rejecting the RWH is essential: doing so would imply that the crude oil market is not fully efficient and that returns are, to some degree, predictable.

Lo and MacKinlay (1999) developed a statistical framework to test and reject the RWH in U.S. equity markets by showing that return series are autocorrelated. Using a comprehensive variance ratio test on weekly returns of the CRSP–NYSE–AMEX index, they found strong positive autocorrelation of about 30 percent, leading to a rejection of the RWH. By analogy, demonstrating statistically significant autocorrelation in crude oil return series would similarly indicate predictability in this market, justifying the application of forecasting models.

1.2. The Influence of the Russian-Ukraine War on Crude Oil Price: This study investigates the quantitative and qualitative impacts of the Russia–Ukraine war on crude oil prices and volatility. Beyond statistical analysis, a review of the literature provides further insights into the mechanisms driving market fluctuations. Pal (2023) argues that Russia's invasion of Ukraine triggered sanctions on Russian oil exports, leading to a sharp reduction in supply. This imbalance between supply and demand contributed to rising prices and heightened volatility, a situation exacerbated by Saudi Arabia's subsequent decision to cut production. At the same time, many countries increased oil purchases to build reserves against future uncertainty, further amplifying both demand and volatility.

The behavior of crude oil prices can be examined across three distinct phases of the conflict. In the pre-war period, prices were shaped by traditional factors such as OPEC policies, global economic conditions, and underlying supply–demand dynamics. During the early stages of the war, however, prices surged above $100 per barrel amid concerns over supply disruptions, with volatility reaching exceptional levels. In the current period, despite international efforts to stabilize markets, crude oil prices remain elevated relative to pre-war levels and continue to be influenced by geopolitical developments.

Ozili (2024) identifies several additional channels through which the conflict has intensified volatility. Sanctions imposed by Western countries—including the United States' ban on Russian oil and gas imports and the European Union's prohibition on exporting refining technologies to Russia—further constrained Russian supply. Russia's restrictions on maritime and air traffic through its territory disrupted supply chains, while precautionary hoarding by firms increased demand pressures. Logistical challenges also arose, such as restrictions on commercial flights near the war zone and enhanced security protocols in neighboring countries, which delayed shipments and raised transportation costs. Together, these factors contributed to both the rise in crude oil prices and the persistence of extreme volatility during the war.

1.3. Statistical Modeling in Crude-Oil Markets: To construct a comprehensive methodology, this study draws on a wide body of academic literature applying ARMA–GARCH and related econometric models to crude oil markets. These works not only provide methodological frameworks but also offer empirical insights into volatility dynamics under different market conditions.

A key reference for our methodology is Loh et al. (2022), who modeled WTI and Brent crude oil returns from January 4, 2010, to July 30, 2021, using conditional mean and variance models. Their analysis of COVID-19's impact revealed that the MA(1)–GJR(1,1) model best captured WTI volatility, while the MA(2)–GJR(1,1) was optimal for Brent. Both models assumed *t*-distributed residuals. Descriptive statistics showed heightened volatility during the pandemic, with higher standard deviations, wider return ranges, and non-normal return distributions characterized by negative skewness and excess kurtosis.

Risk assessments using Value at Risk (VaR) and Expected Shortfall (ES) were conducted through historical simulation, variance–covariance, and ARMA–GARCH approaches.

Other studies provide complementary evidence. Ariyanti and Yusnitasari (2023) applied ARIMA models to daily crude oil prices from January 2020 to January 2023, finding ARIMA(0,1,0) to be optimal with low error metrics (RMSE = 0.01905%). Similarly, Hidayat and Wiharja (2024) modeled WTI monthly prices from April 2019 to April 2023, concluding that ARIMA(1,2,1) yielded accurate 12-month forecasts with a MAPE of 6.81 percent, indicating strong predictive performance. Gasper and Mbwambo (2023) also employed the Box–Jenkins methodology on Brent monthly prices (2002–2022), identifying ARIMA(0,1,1) as the best-fit model based on AIC, BIC, and residual diagnostics.

Beyond ARIMA approaches, several studies integrate GARCH-family models. Anastasiadis and Siskos (2023) evaluated ARMA–GARCH, ARMA–EGARCH, and ARMA–FIGARCH for WTI monthly returns (1980–2022). Their results favored the ARMA–EGARCH(1,20), which achieved the lowest information criteria values and generated accurate three-month forecasts, with only minor deviations from observed returns. Musetescu et al. (2022) also applied GARCH variants—including GARCH(1,1), EGARCH(1,1), and GARCH-M(1,1)—to Brent returns (1987–2022). EGARCH(1,1) was most effective in capturing volatility clustering, while GARCH-M(1,1) provided superior conditional volatility forecasts during 2020–2022, supported by lower RMSE, MAE, and MAPE values. Their findings further showed fat-tailed and positively skewed return distributions, consistent with Jarque–Bera test results rejecting normality.

Studies have also compared competing volatility frameworks. Zhang et al. (2023) assessed the forecasting performance of the OVX index, stochastic volatility models, and GARCH-type models for WTI and Brent (2010–2022). Their results suggest that GARCH-based models, particularly GJR-GARCH, outperform stochastic volatility models for both markets. Additionally, the OVX index offered valuable predictive power for Brent volatility. Similarly, Maiti et al. (2023) compared ARIMA, TARMA, and ENNReg models in forecasting Brent during wartime. Their findings demonstrated that ENNReg achieved the best predictive accuracy, with the lowest RMSE, MAE, and Theil U1 values. Importantly, including a war dummy variable improved model performance, underscoring the significance of geopolitical events in forecasting accuracy.

Taken together, these studies highlight three consistent findings: (1) ARIMA and GARCH-family models are widely effective for capturing crude oil price dynamics; (2) returns frequently deviate from normality, exhibiting fat tails and skewness; and (3) model accuracy improves when contextual shocks—such as COVID-19 or the Russia–Ukraine war—are incorporated. These insights form the empirical and methodological foundation for the present research.

1.4. Market Risk Assessment of the Crude Oil Market: The final strand of our literature review examines market risk assessment in crude oil markets, with a focus on Value at Risk (VaR) and Expected Shortfall (ES/CVaR). Toğuç and Karalınç (2023) provide a comparative analysis of three approaches: linear parametric VaR assuming normality, Monte Carlo Simulation–based VaR (MCS-VaR), and Conditional VaR (ES). Their study uses daily Brent crude oil prices from July 18, 2018, to May 27, 2022, a period encompassing the sharp disruptions triggered by the Russia–Ukraine war.

Consistent with prior research, they found crude oil returns to be positively skewed and fat-tailed, deviating significantly from normality. This casts doubt on the suitability of the conventional linear VaR framework, which relies on normal distributional assumptions. At the 95 percent confidence level, the estimates were −4.86% for Linear VaR, −6.06% for CVaR, and −8.04% for MCS-VaR. At the 99 percent level, the corresponding values were −6.91%, −7.74%, and −9.98%.

Model performance was validated through backtesting. The Kupiec Proportion of Failure (POF) test showed that linear normal VaR consistently underestimated risk at the 95 percent level. By contrast, the Acerbi–Szekely test confirmed that CVaR produced accurate estimates without systematic under- or overestimation, demonstrating its reliability across multiple risk thresholds.

Overall, these findings suggest that normal parametric VaR is ill-suited for modeling the heavy-tailed behavior of Brent returns, leading to underestimation of extreme losses. CVaR, by contrast, offers a more coherent and robust risk measure, while Monte Carlo simulation enhances estimation by incorporating a wider distribution of potential market scenarios.

2. Methodology

The primary objective of this research is to analyze crude oil market behavior using MATLAB. The analysis proceeds in three stages. First, we examine the impact of the Russia–Ukraine war on crude oil price volatility and explore overall market behavior through descriptive statistics. Second, we develop and forecast a range of conditional mean and volatility models for crude oil returns under wartime conditions. Finally, we conduct a comprehensive market risk assessment to evaluate current downside risks in the crude oil market. This section is organized as follows: the first subsection details the step-by-step procedure of our analysis, while the second subsection introduces the key statistical equations and tools employed in the study.

2.1. Procedure of Analysis: The dataset employed in this study consists of daily Brent crude oil spot prices spanning July 1, 2020, to May 19, 2025. Brent serves as the key global oil benchmark, as roughly two-thirds of internationally traded crude oil is priced with reference to it. Data were obtained from the U.S.

Energy Information Administration (EIA). For the purposes of analysis, the dataset was divided into a training set (July 1, 2020–April 3, 2025; 1,206 observations) and a validation set (April 4, 2025–May 19, 2025; 29 observations). All model estimation and selection are conducted on the training set, while the validation set is reserved for forecast evaluation.

The analysis proceeds in several stages. First, descriptive statistics are calculated for the return series to characterize market behavior and assess the influence of the Russia–Ukraine war on crude oil price volatility. Price data are transformed into continuously compounded returns using logarithmic differencing. The return series is further divided into two subperiods—pre-war (July 1, 2020–February 23, 2022) and wartime (July 1, 2020–April 3, 2025)—to evaluate the impact of the conflict. For both periods, descriptive measures such as mean, median, maximum, minimum, standard deviation, skewness, and kurtosis are reported. The Jarque–Bera test is applied to assess normality, and graphical analyses including histograms and QQ-plots are produced to illustrate distributional features.

Next, the Box–Jenkins methodology is applied to identify suitable conditional mean and variance models. The approach involves four stages: model identification, estimation, diagnostic verification, and forecasting. Specifically, ARMA–GARCH family models are considered, with varying AR and MA orders, alternative residual distributions (normal and Student's *t*), and different GARCH specifications [GARCH(1,1), EGARCH(1,1), and GJR(1,1)].

Before model estimation, several statistical tests are conducted. The Ljung–Box Q test is applied to detect autocorrelation in returns, allowing for rejection of the Random Walk Hypothesis. The Augmented Dickey–Fuller (ADF) test is used to confirm stationarity of the series, while Engle's ARCH test evaluates the presence of ARCH effects. Models are then estimated using Maximum Likelihood Estimation (MLE). Candidate models are evaluated based on Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values, with lower values indicating better fit. Diagnostic checks—including the Ljung–Box Q test and the ARCH test on residuals—are performed to verify adequacy. Models with insignificant terms or diagnostic failures are eliminated. The best-fitting model is selected as the one with the lowest AIC and BIC among the remaining candidates.

Both pre-war and wartime return series are fitted with the selected model to analyze parameter changes and shifts in volatility dynamics. Forecasting is then conducted over the 29-day validation period using all models that passed diagnostic checks. Forecast accuracy is assessed using Mean Absolute Error (MAE) and Mean Squared Error (MSE). The model with the lowest error measures is identified as the best forecasting model. Forecasted return and volatility series are plotted for additional analysis.

Finally, market risk is assessed through one-day-ahead Value at Risk (VaR) and Expected Shortfall (ES) on April 4, 2025 (the day following the last training observation), based on a hypothetical $1,000,000 investment. VaR is estimated using three methods: historical simulation, variance–covariance, and ARMA–GARCH. ES is computed through three approaches: historical simulation, parametric ES under normal distribution, and parametric ES under Student's t distribution. A detailed description of these measures and methods is presented in the following subsection.

2.2. Value-at-Risk and Expected Shortfall: Value at Risk (VaR) is one of the most widely used measures in financial risk management. It estimates the potential maximum loss of an asset or portfolio over a given time horizon at a specified confidence level. In other words, it answers the question: *"What is the maximum loss we can expect, with a certain level of confidence, over a defined period?"* For example, a one-day 95% VaR of $1 million implies that there is a 95 percent probability that losses will not exceed $1 million in a single day. VaR can be estimated using several approaches:

Historical Simulation: The historical simulation method is a non-parametric approach that directly relies on past return data without making assumptions about the distribution of returns. A fixed window of historical returns (e.g., 250 trading days) is ordered from smallest to largest, and the quantile corresponding to the desired confidence level is taken as the VaR estimate. For instance, the 5th percentile of historical returns corresponds to the 95% VaR. This method is straightforward and model-free, but it assumes that historical return patterns will adequately represent future risks.

Variance–Covariance (Parametric) Method: Also known as the parametric method, this approach assumes that returns are normally distributed. VaR is computed using the mean (μ), standard deviation (σ), and the appropriate quantile (z_α) from the standard normal distribution. For a portfolio with mean return μ and volatility σ, the one-day VaR at confidence level α is:

$$VaR_\alpha = -\mu + z_\alpha \cdot \sigma$$

where z_α is the critical value from the standard normal distribution (e.g., $z_{0.95} \approx 1.645$). Although computationally efficient, this method often underestimates risk since financial returns typically exhibit fat tails and skewness, deviating from normality.

ARMA–GARCH Approach: This applies the one-step-ahead forecasts of the conditional mean ($\hat{r}_t(1)$) and conditional variance ($\hat{\sigma}_t^2(1)$) from the ARMA-GARCH model fitted to the return series to calculate the one-day Value at Risk (VaR). If the model assumes that residuals follow a standard normal distribution, then the VaR at confidence level α is given by:

$$VaR_\alpha = -(\hat{r}_t(1)) + z_{\alpha,y} \cdot (\hat{\sigma}_t(1))$$

where z_α is the critical value from the standard normal distribution corresponding to confidence level α (e.g., $z_{0.95} \approx 1.645$). If instead the residuals are assumed to follow a standardized Student's t-distribution with degrees of freedom v, then the VaR becomes:

$$VaR_\alpha = -(\hat{r}_t(1)) + t_{\alpha,v} \cdot (\hat{\sigma}_t(1)) \cdot \sqrt{\frac{v-2}{v}},$$

where $t_{\alpha,v}$ is the α-quantile of the standard Student's t-distribution with v degrees of freedom, and the correction factor $(v-2)/v$ ensures the residuals have unit variance.

Expected Shortfall (ES): Also referred to as Conditional Value at Risk (CVaR), Expected Shortfall is a risk measure that estimates the *average loss incurred when losses exceed the Value at Risk (VaR) threshold* at a given confidence level. Unlike VaR, which only identifies a cutoff point for potential losses, ES captures the severity of losses in the tail of the distribution. As a coherent risk measure, ES addresses key limitations of VaR—particularly its inability to reflect tail risk—making it more effective for assessing extreme financial risks. ES can be estimated using either a historical simulation approach or a parametric approach, depending on the distributional assumptions applied to returns.

The historical simulation method is a non-parametric approach that estimates ES directly from past return data. Once an adequately large sample of returns is collected, the values are sorted in ascending order. At a given confidence level (e.g., 95%), ES is computed as the average of all returns that fall below the corresponding Value at Risk (VaR) threshold—in this case, the worst 5% of losses. Because it makes no assumptions about the underlying return distribution, this method is simple and transparent. However, it implicitly assumes that historical return patterns are a reliable representation of future risks.

Parametric Expected Shortfall: The parametric approach to Expected Shortfall (ES) is a model-based method that estimates the average loss beyond the Value at Risk (VaR) threshold under specific distributional assumptions for asset returns. If returns are assumed to follow a normal distribution, ES at confidence level α is given by:

$$ES_\alpha^{Normal} = -\mu + \sigma \cdot \frac{\phi(z_\alpha)}{1-\alpha},$$

where μ is the mean return, σ is the standard deviation, z_α is the standard normal quantile at confidence level α, and $\phi(z_\alpha)$is the standard normal probability density function evaluated at z_α.

If returns are instead assumed to follow a Student's t-distribution with v degrees of freedom, ES is expressed as:

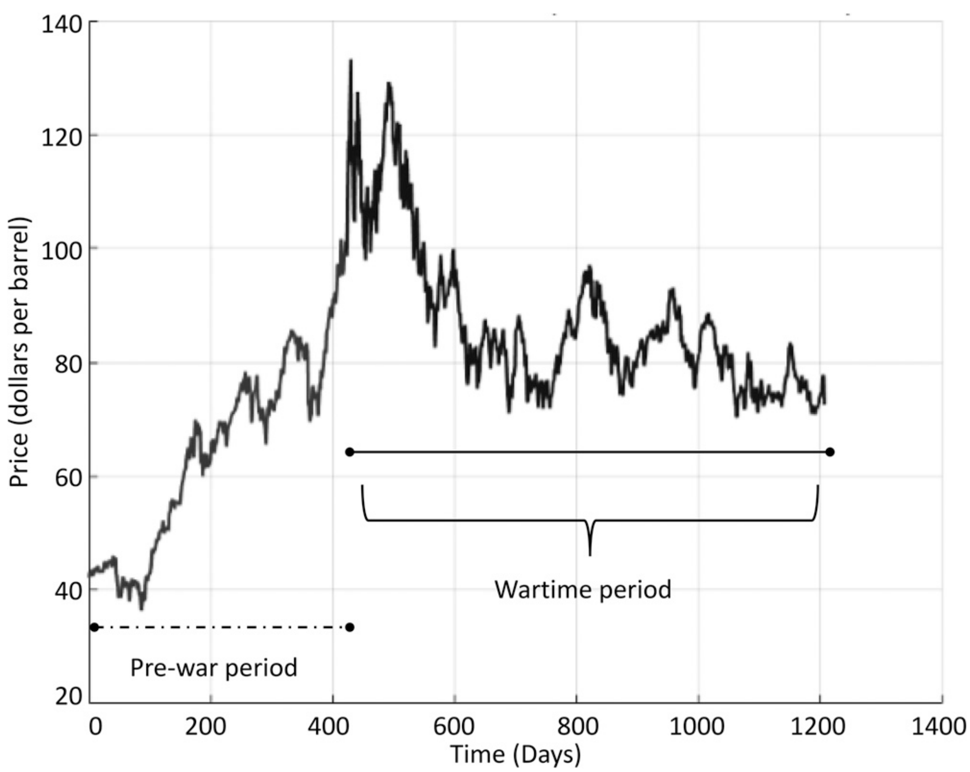
$$ES_\alpha^t = -\mu + \sigma \cdot \frac{f_t(t_{\alpha,v})}{1-\alpha} \cdot \frac{\nu + t_{\alpha,\nu}^2}{\nu - 1},$$

where $t_{\alpha,v}$ is the α-quantile of the Student's t-distribution, with v degrees of freedom, and $f_t(t_{\alpha,v})$ is the probability density function of the t-distribution evaluated at $t_{\alpha,v}$. This parametric framework allows ES to account for different distributional characteristics of returns; in particular, the *t*-distribution better captures fat tails and extreme market risks compared to the normal assumption.

3. Empirical Study

Figure 1 presents the Brent crude oil price series from July 1, 2020, to April 3, 2025, divided into two subperiods: the pre-war period (July 1, 2020–February 23, 2022) and the wartime period (February 24, 2022–April 3, 2025). During the pre-war period, prices rose steadily from approximately \$40 to \$100 per barrel, reflecting, in part, the global economic recovery following the COVID-19 lockdowns. In contrast, the wartime period is marked by a sharp surge in prices immediately after the conflict began, with Brent briefly exceeding \$120 per barrel due to heightened geopolitical tensions and supply disruptions. After this peak, prices remained volatile but trended downward overall, as markets adjusted and policy measures were introduced.

Figure 1
DAILY BRENT CRUDE OIL PRICE FROM JULY 1, 2020, TO APRIL 3, 2025



As shown in Figure 1, the price series is non-stationary, exhibiting time-varying mean and variance. To address this, the data are transformed into continuously compounded returns, displayed in Figure 2, which ensures stationarity and suitability for econometric modeling.

Figure 2 indicates that the return series is stationary, with volatility clustering evident throughout the plot. In the early stages of the war, sharp spikes in both positive and negative returns highlight periods of heightened volatility, followed by intervals of relatively subdued fluctuations. This pattern suggests that large shocks tend to be followed by large shocks, while small shocks are followed by small shocks. Such alternating phases of high and low volatility are characteristic of conditional heteroskedasticity, thereby supporting the application of GARCH-family models for further analysis.

Figure 3 presents the volatility plot, which provides clear evidence of volatility clustering. During the pre-war period, volatility remains relatively low and stable, with only occasional moderate spikes. One notable exception occurs on November 26, 2021, when volatility surged sharply in response to the emergence of the

Figure 2
DAILY BRENT CRUDE OIL RETURN FROM JULY 1, 2020, TO APRIL 3, 2025

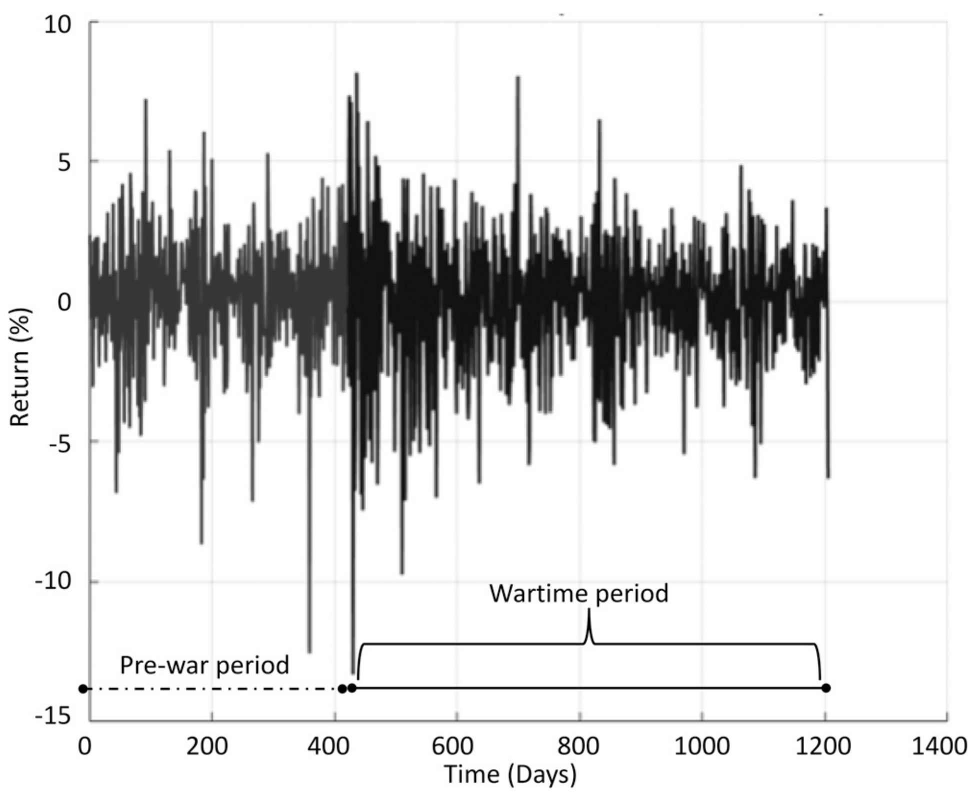
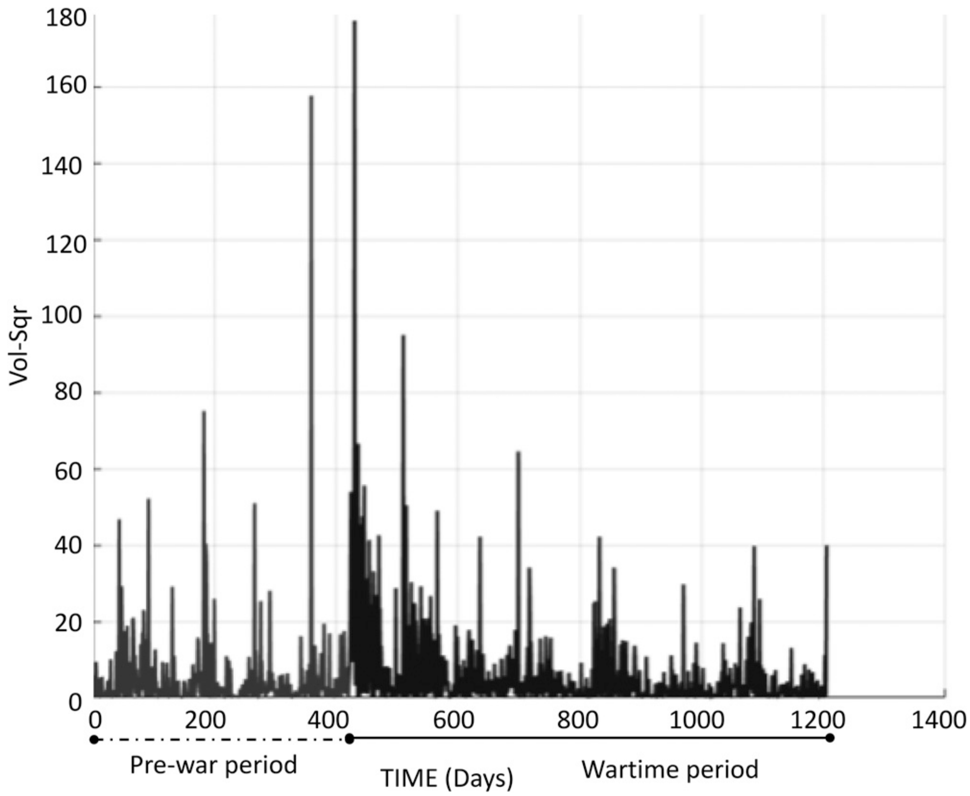


Figure 3
VOLATILITY SERIES OF BRENT CRUDE OIL FROM JULY 1, 2020, TO APRIL 3, 2025



Omicron COVID-19 variant. This event heightened market uncertainty and led to a steep drop in Brent prices.

At the onset of the war, volatility increased dramatically, reflecting the immediate market shock caused by the Russia–Ukraine conflict. This heightened volatility persisted for a period before gradually subsiding, consistent with the well-documented pattern whereby periods of high volatility tend to follow one another. Such clustering behavior supports the application of GARCH-family models to capture the time-varying nature of crude oil market risk.

Table 1 summarizes the descriptive statistics of Brent returns across two distinct periods: pre-war (July 1, 2020–February 23, 2022) and wartime (July 1, 2020–April 3, 2025). In both periods, the average daily return is positive, indicating overall gains in the crude oil market. However, the magnitudes of the maximum and minimum returns are substantially greater during the war, reflecting increased volatility. Specifically, the maximum return occurred on March 17, 2022, when Brent prices rose from approximately \$104.61 to \$113.50 per barrel. Conversely, the

Table 1
DESCRIPTIVE STATISTICS OF BRENT CRUDE OIL RETURN SERIES

Statistic	Before the War (July 1, 2020 to February 23, 2023)	During the War (February 24, 2023 – April 3, 2025)
Number of Observations	420	1205
Mean (%)	0.20383	0.044995
Median (%)	0.38642	0.17843
Standard Deviation (%)	2.1238	2.2393
Maximum (%)	7.2174	8.1564
Minimum (%)	−12.5537	−13.3124
Skewness	−0.83343	−0.57317
Kurtosis	6.9437	5.8677
Jarque-Bera Test Statistic	320.7899	478.8807
JB Critical Value (5% level)	5.8326	5.9410
JB p-value	0.001	0.001
Normality Test Result (JB Test)	Reject Normality	Reject Normality

minimum return was recorded on March 9, 2022, when prices dropped from about \$133.18 to \$116.58 per barrel, corresponding to a one-day loss of 13.31 percent.

This heightened variability is further confirmed by the higher standard deviation observed during the wartime period compared to the pre-war period. A larger standard deviation indicates greater dispersion of returns, underscoring the elevated price risk faced by the crude oil market during the conflict.

The skewness of Brent returns is negative in both the pre-war and wartime periods, indicating a left-skewed distribution. Moreover, the kurtosis values for both periods are well above 3, suggesting leptokurtic distributions with fat tails. Together, the negative skewness and elevated kurtosis confirm that Brent returns deviate substantially from normality.

To formally test this, we apply the Jarque–Bera test to the return series for both periods. The hypotheses of the test are as follows:

- Null hypothesis (H_0): The return series follows a normal distribution.
- Alternative hypothesis (H_1): The return series does not follow a normal distribution.

Given the very small p-values for both periods, we reject the null hypothesis and conclude that Brent crude oil returns are not normally distributed. This finding aligns with the results reported by Toğuç and Karalınç (2023), Musetescu et al. (2022), and Zhang et al. (2023).

3.1. Modeling and Forecasting Brent Returns: In this section, we develop the most suitable model for analyzing the behavior of crude oil returns during the war period and for forecasting future returns and volatility. The training dataset covers Brent returns from July 1, 2020, to April 3, 2025, and is used to estimate the ARMA–GARCH model. A key objective of this modeling exercise is to determine whether Brent returns are predictable and whether conditional volatility can be forecasted.

To verify that the series does not follow a random walk, we first apply the Ljung–Box Q test to the training set. The null hypothesis H_0 assumes no autocorrelation (independent distribution), while the alternative H_1 posits the presence of autocorrelation at one or more lags. As reported in Table 2, the test results reject the null hypothesis at all lags (p-values < 0.05), indicating significant serial correlation. These results confirm that Brent returns do not follow a random walk and are therefore forecastable. The presence of autocorrelation also justifies the use of an ARMA model, which is specifically designed to capture such dependence structures in time series.

Next, we assess volatility dynamics using Engle's ARCH test and evaluate stationarity with the Augmented Dickey–Fuller (ADF) test. The ARCH test examines the null hypothesis of no autoregressive conditional heteroskedasticity (homoscedasticity) against the alternative of ARCH effects (heteroskedasticity). In parallel, the ADF test checks for a unit root, with the null hypothesis H_0 of non-stationarity and the alternative H_1 of stationarity. The results of these two diagnostic tests are presented in Table 3.

Based on the results of the ARCH test, the Brent return series exhibits significant ARCH effects, confirming the presence of heteroskedasticity. This finding supports the use of GARCH-family models, which are designed to capture time-varying volatility. In addition, the ADF test result indicates that the series is stationary, thereby validating the application of ARMA models to the Brent return data.

Following the procedure outlined in the methodology section, we evaluated several candidate models for suitability. Four models emerged as strong candidates,

Table 2
LJUNG-BOX Q TEST RESULTS FOR BRENT RETURN SERIES

Lag	Q-statistic	Critical Value (Q_{crit})	p-value	Reject H_0?
6	18.5907	12.5916	0.0049	Yes
12	25.1941	21.0261	0.0139	Yes
18	35.6957	28.8693	0.0077	Yes

Table 3
ARCH AND ADF TEST RESULTS FOR BRENT RETURN SERIES

Test	Lag/Order	Test Statistic	Critical Value	p-value / Decision
ARCH	6	101.8272	12.5916	$p < 10^{-15}$, Reject H_0
ARCH	12	105.7314	21.0261	$p < 10^{-15}$, Reject H_0
ARCH	18	117.0614	28.8693	$p = 1.11 \times 10^{-14}$, Reject H_0
ADF	–	−34.1563	−1.9416	$p = 0.001$, Reject H_0

Table 4
MODEL COMPARISON WITH AIC/BIC RANKINGS AND TOTAL SCORE

Model	AIC	AIC Rank	BIC	BIC Rank	Total Score
ARMA(1,1)–GJR(1,1)	5166.3	2	5207.0	4	6
ARMA(1,1)–EGARCH(1,1)	5165.6	1	5206.3	3	4
ARMA(0,0)–GJR(1,1)	5168.1	3	5198.6	2	5
ARMA(0,0)–GARCH(1,1)	5170.3	4	5195.7	1	5

each demonstrating low AIC or BIC values, statistically significant parameters, and satisfactory performance on residual diagnostic tests (ARCH and Ljung–Box Q). The selected models are: ARMA(1,1)–GJR(1,1), ARMA(1,1)–EGARCH(1,1), ARMA(0,0)–GJR(1,1), and ARMA(0,0)–GARCH(1,1). All models are estimated under the assumption of Student's t-distributed residuals. The corresponding AIC and BIC values are reported in Table 4.

To identify the best-fitting model for the Brent return series, we employed a ranking-based scoring system using both AIC and BIC values. Each model was assigned a rank from 1 (best) to 4 (worst) for each criterion, and the total score was calculated as the sum of its AIC and BIC ranks. The model with the lowest total score was selected as the most appropriate, as this indicates the best trade-off between model fit and parsimony.

According to this ranking procedure, the ARMA(1,1)–EGARCH(1,1) model with Student's t-distributed residuals emerged as the best-fitting specification. We then estimated this model for Brent crude oil returns across two sub-periods—during the war and before the war—to analyze potential parameter shifts. The estimation results are presented in the following pages: Table 5 reports the ARMA(1,1)–EGARCH(1,1) estimates for the war period, while Table 6 provides the corresponding estimates for the pre-war period.

Table 5
ARMA(1,1)–EGARCH(1,1) MODEL ESTIMATES (STUDENT'S T-DISTRIBUTION) FOR BRENT CRUDE OIL RETURNS DURING THE WAR (JULY 1, 2020–APRIL 3, 2025)

Parameter	Estimate	Std. Error	T-Statistic	P-Value
		Mean Equation (ARMA(1,1))		
Constant	0.24541	0.09982	2.4586	0.01395
AR(1)	-0.82097	0.08079	-10.162	2.93×10^{-24}
MA(1)	0.85764	0.07218	11.882	1.47×10^{-32}
Degree of Freedom	6.7386	1.1134	6.052	1.4306×10^{-9}
		Variance Equation (EGARCH(1,1))		
Constant	0.06586	0.02401	2.7433	0.00608
GARCH(1)	0.95384	0.01579	60.402	0
ARCH(1)	0.19717	0.03730	5.2856	1.25×10^{-7}
Leverage(1)	-0.06160	0.02632	-2.3404	0.01927

Table 6
ARMA(1,1)–EGARCH(1,1) MODEL ESTIMATES (STUDENT'S T-DISTRIBUTION) FOR BRENT CRUDE OIL RETURNS BEFORE THE WAR (JULY 1, 2020–FEBRUARY 23, 2022)

Parameter	Estimate	Std. Error	T-Statistic	P-Value
Mean Equation (ARMA(1,1))				
Constant	0.14295	0.08261	1.7305	0.08353
AR(1)	0.53287	0.24989	2.1324	0.03298
MA(1)	−0.62692	0.22733	−2.7577	0.00582
Degree of Freedom	5.6002	1.3405	4.1778	2.9431×10^{-5}
Variance Equation (EGARCH(1,1))				
Constant	0.14575	0.07185	2.0285	0.04251
GARCH(1)	0.88786	0.05086	17.456	3.12×10^{-68}
ARCH(1)	0.15491	0.08627	1.7957	0.07255
Leverage(1)	−0.14674	0.05820	−2.5211	0.01170

The ARMA(1,1)-EGARCH(1,1) Student's t-distribution model for Brent daily return during the war is:

Mean Equation: $r_t = 0.24541 - 0.82097 r_{t-1} + \varepsilon_t + 0.85764 \varepsilon_{t-1}$

$$\text{Variance Equation: } log\left(\sigma_t^2\right) = 0.06586 + 0.19717\left(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}}\right) - 0.0616\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right) + 0.95384 log\left(\sigma_{t-1}^2\right)$$

The ARMA(1,1)-EGARCH(1,1) Student's t-distribution model for Brent daily return before the war is:

Mean Equation: $r_t = 0.14295 - 0.53287 r_{t-1} + \varepsilon_t - 0.62692 \varepsilon_{t-1}$

$$\text{Variance Equation: } log\left(\sigma_t^2\right) = 0.14575 + 0.15491\left(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}}\right) - 0.14674\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right) + 0.88786 log\left(\sigma_{t-1}^2\right)$$

Before the war, the Brent crude oil market exhibited moderately persistent volatility, as indicated by the GARCH coefficient of 0.88786 in the EGARCH model. The ARCH coefficient (0.15491) suggests that returns reacted modestly to recent shocks, while the negative leverage effect (–0.14674) shows that adverse news events disproportionately increased volatility. Taken together, these results imply that although the market was sensitive to negative shocks, volatility tended to dissipate relatively quickly, contributing to a more stable risk environment overall.

During the war, the volatility dynamics of the Brent crude oil market shifted markedly. The GARCH coefficient rose to 0.95384, indicating greater persistence in volatility, with shocks taking longer to dissipate. The ARCH term also increased to 0.19717, suggesting heightened sensitivity to recent shocks. Interestingly, the leverage effect weakened (–0.06160), implying a less asymmetric response between positive and negative shocks. Overall, the higher persistence and shock responsiveness reflect a riskier, more unpredictable environment during the war, consistent with the influence of geopolitical tensions on market behavior.

Contrary to expectations, the volatility constant declined from 0.14575 (pre-war) to 0.06586 during the war. One plausible explanation is the residual impact of the COVID-19 pandemic, which had already elevated volatility due to sharp demand declines. The higher pre-war constant may capture this baseline uncertainty, while the lower constant during the war suggests that although volatility became more persistent and reactive to shocks, the baseline level had normalized in the post-pandemic period.

Building on these findings, we apply the four best-fitting models identified earlier to forecast Brent returns from April 4, 2025, to May 19, 2025. Forecasted returns are compared with actual values in the validation set, and performance is assessed using Mean Absolute Error (MAE) and Mean Squared Error (MSE). The model with the lowest MAE and MSE is selected as the most accurate forecasting tool for current crude oil market conditions. The MAE and MSE values for each model are reported in Table 7.

The results indicate that the ARMA(1,1)–GJR(1,1) model provides the most accurate forecasts for current crude oil returns. Notably, this differs from the best-fitting model identified in the previous section, highlighting that the model with the best in-sample fit is not always the most effective for out-of-sample forecasting. The forecasting model can be expressed as follows:

Mean Equation: $r_t = 0.2363 - 0.8226 r_{t-1} + \varepsilon_t + 0.8602 \varepsilon_{t-1}$

Variance Equation: $\sigma_t^2 = 0.2032 + 0.0583 \varepsilon_{t-1}^2 + 0.0715 \varepsilon_{t-1}^2 I_{\{\varepsilon_{t-1} < 0\}} + 0.8640 \sigma_{t-1}^2$

Next, we analyze the forecasted returns and conditional variance generated by the ARMA(1,1)–GJR(1,1) model. Figure 4 presents the forecasted returns, while

Table 7
FORECAST ACCURACY METRICS FOR EACH MODEL CANDIDATES (STUDENT'S-T RESIDUAL DISTRIBUTION)

Model	MAE	MSE
ARMA(1,1) – GJR(1,1)	2.0507	5.5164
ARMA(1,1) – EGARCH(1,1)	2.0519	5.5234
ARMA(0,0) – GJR(1,1)	2.0642	5.6384
ARMA(0,0) – GARCH(1,1)	2.0668	5.6520

Figure 4
CONDITIONAL RETURN FORECASTS USING ARMA (1,1)-GJR (1,1) STUDENT'S T
(FORECAST PERIOD: APRIL 4, 2025 TO MAY 19, 2025)

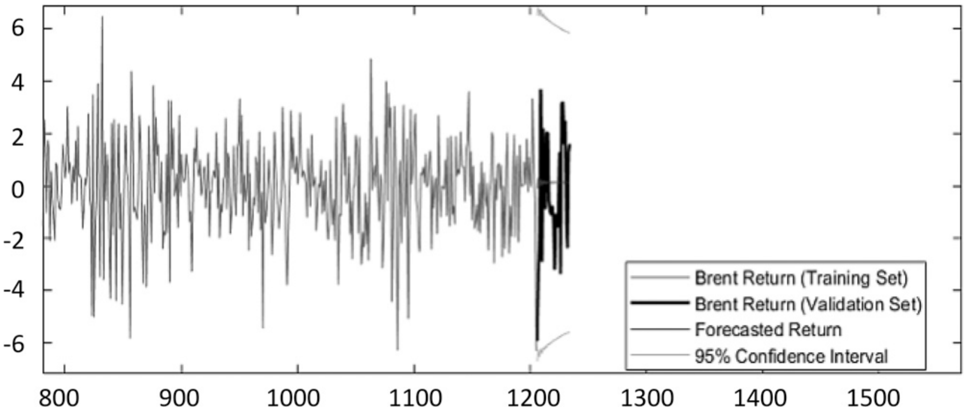
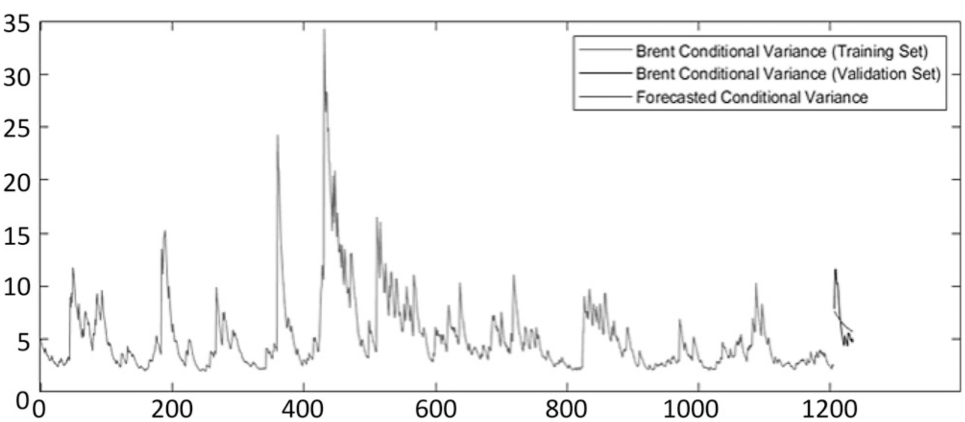


Figure 5 illustrates the forecasted conditional variance. The return forecasts remain stable and close to zero, consistent with the mean-reverting behavior typically observed in financial markets. Moreover, the 95% confidence interval gradually narrows across the forecast horizon, suggesting reduced uncertainty in the projections. Importantly, the validation set falls largely within the forecasted range, confirming the model's ability to capture short-term market dynamics. Overall, these results demonstrate the strong forecasting performance of the ARMA(1,1)–GJR(1,1) model.

In addition, the conditional variance plot demonstrates that the model successfully replicates volatility clustering in the training period and generates a smooth,

Figure 5
CONDITIONAL VARIANCE FORECASTS USING ARMA (1,1)-GJR (1,1) STUDENT'S T
(FORECAST PERIOD: APRIL 4, 2025 TO MAY 19, 2025)



downward-sloping forecast of future volatility. This result is consistent with the narrowing confidence intervals observed in the return forecasts, as lower volatility translates into tighter prediction bounds. The ARMA(1,1)–GJR(1,1) model with Student's t-distributed residuals provides a well-aligned volatility forecast, effectively capturing the overall declining trend in the validation set. This strong performance further supports its suitability for forecasting future crude oil market dynamics.

3.2. Market Risk Assessment of the Current Crude Oil Market: As outlined in the methodology, we assess market risk in the current crude oil market using two key statistical measures: Value at Risk (VaR) and Expected Shortfall (ES). The analysis is conducted on the training return series spanning July 1, 2020, to April 3, 2025, with a focus on estimating the one-day VaR and ES for April 4, 2025, based on an assumed investment of $1,000,000.

VaR is computed using three approaches: historical simulation, variance–covariance, and the ARMA–GARCH framework. ES is derived using both historical simulation and parametric methods. As noted in the methodology section, the ARMA–GARCH VaR is based on one-step-ahead forecasts of returns and conditional variance obtained from our best-fitting model, the ARMA(1,1)–EGARCH(1,1) with Student's t-distributed residuals. The results of the VaR and ES analysis are presented in Tables 8 and 9.

In our analysis, an $X\%$ VaR $= Y$ indicates that for a $1,000,000 investment in Brent crude oil on April 3, 2025, the portfolio loss on the next trading day (April 4, 2025) will not exceed Y with $X\%$ confidence. For instance, under the Historical Simulation approach, the 95% one-day VaR is $37,442.09, meaning there is a 5%

Table 8
VALUE AT RISK (VAR) AT 95% AND 99% CONFIDENCE LEVELS (PORTFOLIO VALUE: $1,000,000)

Method	**VaR (95%)**	**VaR (99%)**
Historical Simulation VaR	37,442.09	65,050.34
Variance-Covariance VaR (Normal)	36,387.33	51,726.87
ARMA-GARCH VaR (Student-t)	39,786.93	62,795.10

Table 9
EXPECTED SHORTFALL (ES) AT 95% AND 99% CONFIDENCE LEVELS (PORTFOLIO VALUE: $1,000,000)

Method	**ES (95%)**	**ES (99%)**
Historical Simulation ES	54,643.72	83,307.44
Parametric ES (Normal Distribution)	45,741.38	59,233.49
Parametric ES (Student-t Distribution)	58,242.12	85,283.75

probability that the loss will exceed this value. Similarly, the 99% VaR of $65,050.34 implies that with 99% confidence, losses will remain below this threshold.

Expected Shortfall (ES) provides additional insight by measuring the average loss beyond the VaR threshold. An X% ES $= Y$ means that, conditional on losses exceeding the X% VaR, the expected loss is Y. For example, the 95% Historical Simulation ES is $54,643.72, which implies that if losses surpass the 95% VaR, the average loss would be $54,643.72. At the 99% level, the Historical Simulation ES rises to $83,307.44.

On April 4, 2025, the actual Brent return was –5.935%, corresponding to a portfolio loss of $59,350 on a $1,000,000 investment—a severe outcome for Brent crude oil. At the 95% level, all models underestimated the realized loss, demonstrating the limitations of lower confidence thresholds in capturing extreme market movements. At the 99% level, both the Historical Simulation and ARMA–GARCH approaches provided conservative estimates that exceeded the actual loss. By contrast, the Variance–Covariance method, based on the normality assumption, underestimated risk, producing a VaR below the realized loss—consistent with findings reported in Toğuç, and Karalınç (2023).

The ES results further confirm this pattern. At 99%, both the Historical Simulation ES and the parametric ES under the Student's t-distribution exceeded the realized loss, demonstrating their ability to capture tail risk. In contrast, the normal-based parametric ES underestimated the severity of losses, reflecting its inability to accommodate fat tails and skewness in return distributions.

The key takeaway for investors is that reliance on low confidence levels (e.g., 95%) can create a false sense of security, as these thresholds fail to capture rare but severe events. Even at higher levels, models assuming normality remain inadequate. By contrast, historical simulation and Student's t-based approaches provide more reliable risk estimates by incorporating fat tails, thereby offering a more realistic assessment of market risk. These findings underscore the importance of using higher confidence levels and robust fat-tailed models to strengthen portfolio protection against extreme losses.

4. Conclusion

This study investigates the behavior of Brent crude oil returns and the impact of the Russia–Ukraine war on market volatility. The dataset consists of Brent spot prices from July 1, 2020, to April 3, 2025, divided into two sub-periods: pre-war (July 1, 2020 – February 23, 2022) and wartime (July 1, 2020 – April 3, 2025). Descriptive statistics reveal that average daily returns were positive in both periods, reflecting overall price gains. However, volatility rose sharply during the war, as indicated by higher maximum, minimum, and standard deviation values. Both periods exhibit negative skewness and excess kurtosis, confirming that Brent returns are non-normally distributed. The Jarque–Bera test, QQ-plots, and histograms further reject normality and show that the Student's t distribution provides a better fit for return behavior.

To model volatility, we applied the Box–Jenkins methodology and identified four candidate models: ARMA(1,1)–GJR(1,1), ARMA(1,1)–EGARCH(1,1), ARMA(0,0)–GJR(1,1), and ARMA(0,0)–GARCH(1,1), all estimated under Student's t-distributed residuals. Based on AIC and BIC rankings, the ARMA(1,1)–EGARCH(1,1) model was selected as the best-fitting specification. Comparison of pre-war and wartime estimates shows an increase in the GARCH coefficient (0.88786 → 0.95384), indicating more persistent volatility during the war, and an increase in the ARCH term (0.15491 → 0.19717), reflecting stronger sensitivity to recent shocks. The significant leverage effect in both periods highlights the asymmetric impact of negative news on volatility. Together, these findings confirm that the war created a more fragile and unpredictable risk environment.

Next, forecasting performance was evaluated using the four candidate models to predict 29 daily returns from April 4 to May 19, 2025. The ARMA(1,1)–GJR(1,1) model achieved the lowest MAE (2.0507) and MSE (5.5164), making it the best forecasting model. Its return forecasts remained stable and mean-reverting around zero, with confidence intervals narrowing over time. The validation set aligned closely with the forecasted range, demonstrating strong short-term predictive accuracy. The model also replicated volatility clustering during training and produced a smooth, declining variance forecast, confirming its ability to capture the downward trend in conditional volatility. Overall, the ARMA(1,1)–GJR(1,1) with Student's t residuals emerges as a robust tool for forecasting Brent crude oil returns.

The final part of the analysis assesses market risk through Value at Risk (VaR) and Expected Shortfall (ES), calculated for April 4, 2025, with an assumed $1,000,000 investment. Multiple methods were applied, including historical simulation, variance–covariance, and ARMA–GARCH for VaR, and historical and parametric approaches for ES. Results show that at the 95% level, all methods underestimated the realized loss (–5.935%, or $59,350). Even at 99%, the variance–covariance method based on normality failed to capture tail risk. By contrast, historical simulation and Student's t-based models produced estimates exceeding the realized loss, demonstrating their effectiveness in capturing extreme events.

The key implication for investors is that lower confidence levels (e.g., 95%) underestimate severe risks, while normality-based models remain inadequate even at higher thresholds. More robust, fat-tailed approaches—particularly those using the Student's t distribution—offer more reliable estimates of tail risk and should be favored for portfolio protection in volatile energy markets.

Future Directions. To advance this line of research, future studies could incorporate advanced or non-linear volatility models, such as Markov-Switching GARCH, Stochastic Volatility, or Component GARCH, to capture regime shifts, long-memory effects, and structural breaks in oil price dynamics. Machine learning approaches, including LSTM, Random Forest, or XGBoost, may also be explored to evaluate whether data-driven methods improve forecasting accuracy and risk assessment. Finally, integrating macroeconomic and geopolitical variables—such as interest rates, exchange rates, or geopolitical risk indices—could enhance predictive power by accounting for broader drivers of crude oil market behavior beyond past returns.

REFERENCES

Anastasiadis, V., and E. Siskos. 2023. "Time Series Analysis for Forecasting Crude Oil Prices." *Journal of European Economy* 22 (3): 430–54. https://doi.org/10.35774/jee2023.03.430.

Ariyanti, V. P., and T. Yusnitasari. 2023. "Comparison of ARIMA and SARIMA for Forecasting Crude Oil Prices." *Jurnal RESTI (Rekayasa Sistem dan Teknologi Informasi)* 7 (2): 405–13. https://doi.org/10.29207/resti.v7i2.4895.

Gasper, L., and H. Mbwambo. 2023. "Forecasting Crude Oil Prices by Using ARIMA Model: Evidence from Tanzania." *Journal of Accounting, Finance and Auditing Studies* 9 (2): 158–75.

Hidayat, A. S. E., and J. M. Wiharja. 2024. "ARIMA Modeling for Monthly West Texas Intermediate Crude Oil Price Forecasting." *Indonesian Journal of Applied Mathematics and Statistics* 1 (1): 17–26. https://doi.org/10.71385/idjams.v1i1.6.

Loh, A. Y. B., A. K. Chong, W. C. Chin, and M. Lim. 2022. "Crude Oil Market Risk Evaluations under the Presence of the COVID-19 Pandemic." *Journal of Energy and Development* 47 (2): 155–75.

Lo, A. W., and A. C. MacKinlay. 1999. *A Non-Random Walk Down Wall Street.* Princeton, NJ: Princeton University Press.

Maiti, M. 2021. *Applied Financial Econometrics: Theory, Method and Applications.* Palgrave Macmillan.

Mati, S., M. Radulescu, N. Saqib, A. Samour, G. Y. Ismael, and N. Aliyu. 2023. "Incorporating Russia–Ukrainian War in Brent Crude Oil Price Forecasting: A Comparative Analysis of ARIMA, TARMA and ENNREG Models." *Heliyon* 9 (11): e21439. https://doi.org/10.1016/j.heliyon.2023.e21439.

Musetescu, R.-C., G.-E. Grigore, and S. Nicolae. 2022. "The Use of GARCH Autoregressive Models in Estimating and Forecasting the Crude Oil Volatility." *European Journal of Interdisciplinary Studies* 14 (1): 13–38. https://doi.org/10.24818/ejis.2022.02.

Ozili, P. K. 2024. "Global Economic Consequence of Russian Invasion of Ukraine." *SSRN Electronic Journal*. Posted April 20, 2022; last revised May 18, 2025. https://doi.org/10.2139/ssrn.4064770.

Pal, H. 2023. "The Impact of Russia–Ukraine Conflict on Global Commodity Brent Crude Prices." *ScienceOpen Preprints.* Preprint.

Toğuç, N., and T. Karalınç. 2023. "A Practical Risk Analysis of the Brent Oil Prices." *Üçüncü Sektör Sosyal Ekonomi Dergisi* 58 (1): 932–50.

Zhang, Z., M. Y. Raza, W. Wang, and L. Sui. 2023. "Volatility Predictability in Crude Oil Futures: Evidence Based on OVX, GARCH and Stochastic Volatility Models." *Energy Strategy Reviews* 50: 101209. https://doi.org/10.1016/j.esr.2023.101209.